Quantum Hall effect

Consider electrons in a strong magnetic field

In 3D

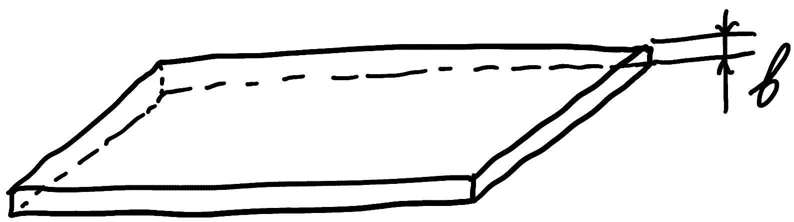
$$\varepsilon(N, k_z) = \hbar\omega\left(N+\frac{1}{2}\right) + \frac{\hbar^2 k_z^2}{2m}, \quad N = 0, 1, 2, \ldots$$

k_z - wavevector along the z axis

Each Landau level is degenerate, with $\gamma = \frac{S}{2\pi l_B^2}$, $l_B = \left(\frac{\hbar c}{|e|B}\right)^{\frac{1}{2}}$,

S - the cross-section area in the direction perpendicular to $\vec{B}$.

Note also that $\gamma = \frac{BS}{\varphi_0}$, $\varphi_0 = \frac{2\pi\hbar c}{|e|}$

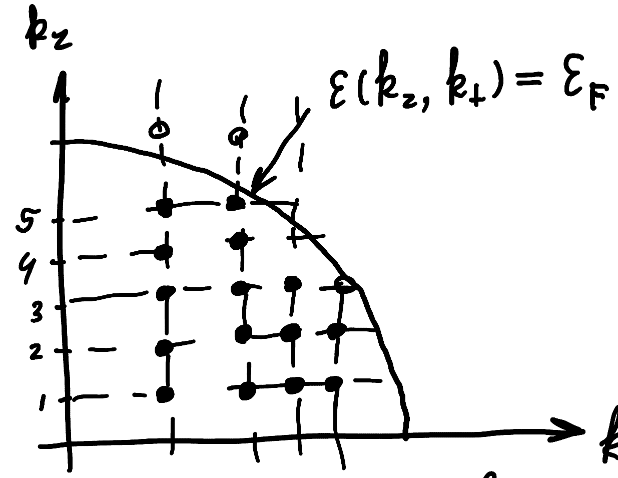


$$k_z \to \frac{\pi}{b} N_z$$

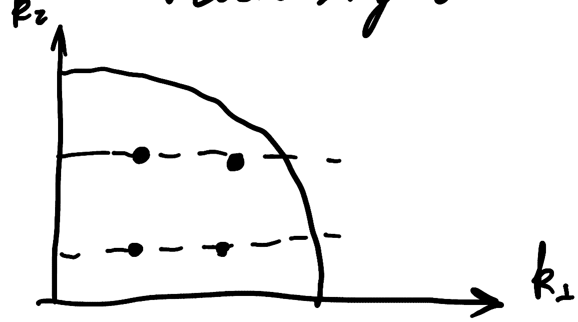
Assume, n_0 is the concentration of electrons in 3D. The number of filled Landau levels:

$$\nu = \frac{n_0 S b}{\gamma} = \frac{2\pi\hbar c\, n_0}{|e|B}$$

$[\nu]$, the integer part of ν, is the number of filled Landau levels



↓ Decreasing B



QHE regime

$\hbar\omega \gg \tau^{-1}$

Increasing B also makes ν decrease

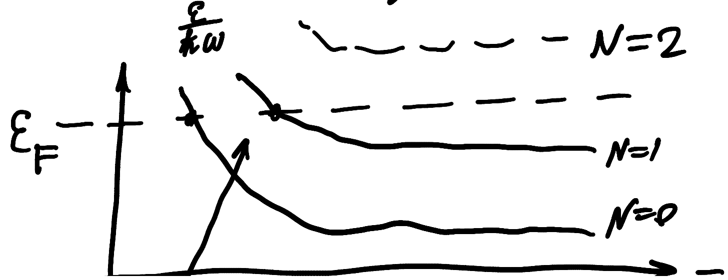
Note, we neglect spin

$-g\,\mu_B B \sim \underbrace{\frac{e\hbar}{2m_e}}_{\text{Bohr magneton}} B$

$\hbar\omega = \frac{\hbar e}{m} B \leftarrow$ effective mass $\ll m_e$ $\rightarrow \hbar\omega \gg \mu_B B$

For each level N there are circular orbits whose radius is $r_N \approx r_B\sqrt{N}$

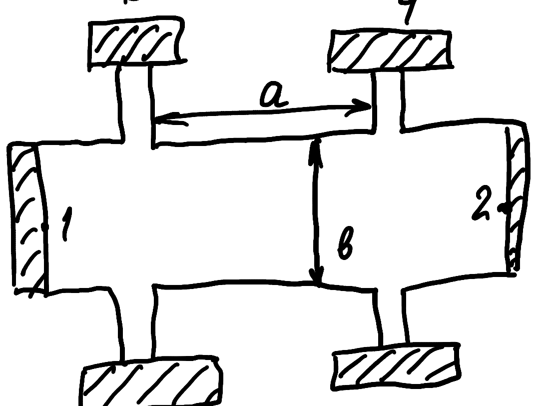
Considering a finite sample means it has an edge



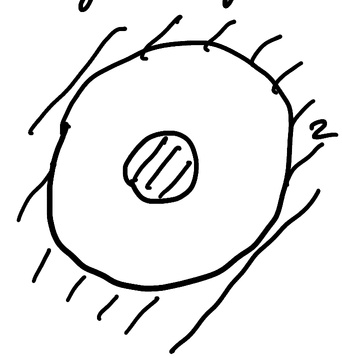
Edge channels

Measurement of Hall conductance

Hall bar



Corbino geometry



$$\rho_{xx} = \frac{V_{34}}{J_{12}} \frac{a}{b}$$

$$\sigma_{xx} = \frac{J_{12}}{2\pi V_{12}} \ln \frac{a_2}{a_1}$$

$$\rho_{xy} = \frac{V_{35}}{J_{12}}$$

Reminder:

$$\begin{cases} j_x = \sigma_{xx} E_x + \sigma_{xy} E_y \\ j_y = \sigma_{yx} E_x + \sigma_{yy} E_y \end{cases}$$

$$\vec{j} = \hat{\sigma} \vec{E}$$

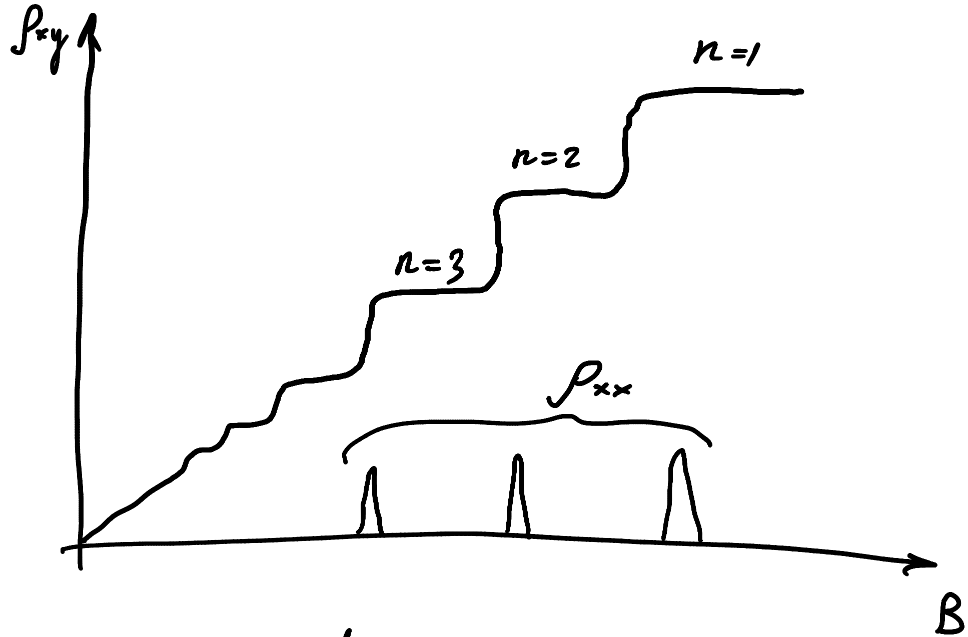
$$\sigma_{xx} = \sigma_{yy}$$

$$\sigma_{xy} = -\sigma_{yx}$$

$$\vec{E} = \hat{\rho} \vec{j}$$

$\sigma_{xy} = $

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \qquad \sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$

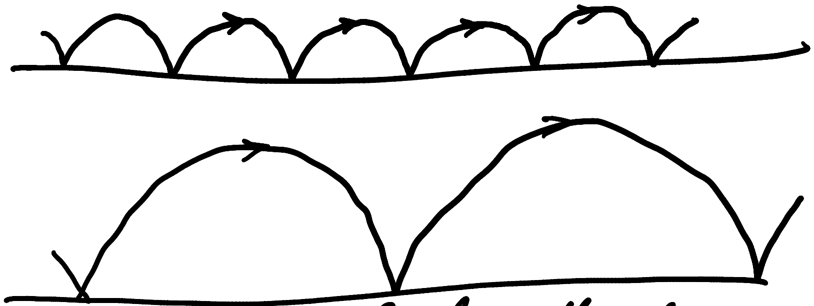


$$\rho_{xy} = \frac{1}{n}\,\frac{2\pi\hbar}{e^2}$$

Naive explanation

$\rho_{xy} = G$ - conductance for edge channels $= \frac{e^2}{2\pi\hbar} \times$ number of channels

That relies on the assumption that there are channels along the edges which carry charge



(Cannot be backscattered)

The states in the bulk do not contribute because they are dispersionless

However, this is too naive because there is disorder

Continuous percolation problem

Consider a big sample with random potential $U(\vec{r})$. Paint with white colour regions $E > U(\vec{r})$, black colour - $E < U(\vec{r})$

We change E.
When does an infinite black cluster appear?

When the fraction of white = fraction of black $= \frac{1}{2}$

There are 2 types of disorder:

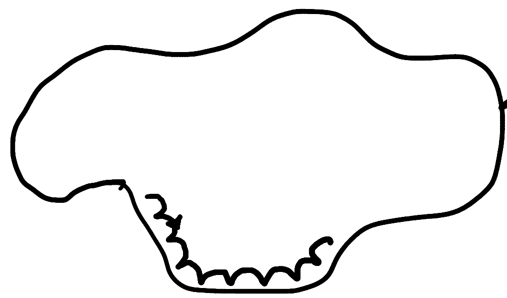
*) short-range, $\xi \ll l_B$

*) long-range, $\xi \gg l_B$ ← focus on this

If there is a region of low potential

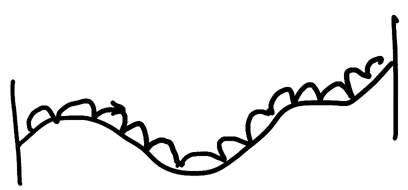
$U < E_F - \hbar\omega(n + \frac{1}{2})$ and outside of

$U < E_F - \hbar\omega(n+\frac{1}{2})$ and outside of this region $U > E_F - \hbar\omega(n+\frac{1}{2})$



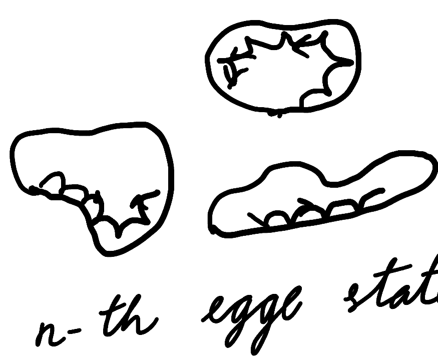
$U = U_n \equiv E_F - \hbar\omega(n+\frac{1}{2})$

Along this region there runs the n-th edge state. So, there exist lots of localised „edge" states. There may also exist delocalised states = infinite cluster



Consider $E_F = \hbar\omega(n+\frac{1}{2}) - \delta$ $(\delta > 0)$

Due to fluctuations of the potential there will be regions with n-th edge state inside



The n-th egge state runs iside finite clusters

Increase the E_F slightly; the clusters get

Increase the E_F slightly; the clusters get „fatter"

When $E_F = \hbar\omega(n+\frac{1}{2}) + \delta \quad (\delta > 0)$

When $E_F \approx \hbar\omega(n+\frac{1}{2})$ there are infinite clusters